A Linear Dependence of $F_{\min}$ on Frequency for FET's

Brian Hughes, *Member, IEEE*

Abstract— **The minimum noise figure of a FET, expressed in decibels, is shown to increase approximately linearly with frequency to frequencies approaching the FET's $f_{\max}$. This observation is useful for extrapolating noise figures and provides a simple frequency-of-merit.**

I. Introduction

NATURE is occasionally kind, and an approximation works much better than expected. Surprisingly, the minimum noise figure of a FET, $F_{\min}$, which is expressed in decibels, increases linearly with frequency to frequencies that approach the $f_{\max}$ of the FET. This simple linear relationship is useful because $F_{\min}$ (the most important noise parameter) is usually measured and presented in decibels. This paper shows experimental evidence that $F_{\min}$ is proportional to frequency, and uses a noise model to explain the linear behavior. $F_{\min}$ results are easily extrapolated versus frequency with the linear approximation. A simple figure-of-merit is $f/F_{\min}$ (dB). This frequency-of-merit can be used to compare noise performance as f_T and $f_{\max}$ are used to compare the gain of different FET's.

II. Approximation

Noise theories predict that the equivalent minimum input noise temperature of a FET ($T_{\min}$) increases linearly with frequency at low frequencies, but increases faster than linear at higher frequencies [1], [2]. In this paper, $T_{\min}$ is modeled with Pospieszalski's resistor temperature noise model [2], [3]. The expression for $T_{\min}$ is

$$T_{\min} = (T_g T_d)^{1/2} \frac{f}{f_{\max}} \left(\left(1 + \frac{T_d}{4T_g} \left(\frac{f}{f_{\max}} \right)^2 \right)^{1/2} + \left(\frac{T_d}{4T_g} \right)^{1/2} \frac{f}{f_{\max}} \right) \tag{1}$$

$$\approx (T_g T_d)^{1/2} \frac{f}{f_{\max}} \qquad f \ll f_{\max}. \tag{1a}$$

$f_{\max}$ is the frequency where the $G_{A\max}$ of the FET extrapolates to unity. T_g and T_d are the equivalent noise temperatures of the input resistor and output resistor of the simple FET equivalent circuit model shown in Fig. 1. T_g has been shown to be close to ambient [2]. T_d of an intrinsic FET, biased for low noise, is typically 1000–2500 K [2], [4], [21]–[23]. The effective T_d of an extrinsic FET noise model is much lower, typically between 300 and 700 K, because the extrinsic $f_{\max}$ is proportionally lower [3]. Extrinsic T_d and $f_{\max}$ values are used in this paper so extrinsic $F_{\min}$'s are directly predicted. The frequency dependence of $T_{\min}$ (1) is shown in Fig. 2. The linear approximation for $T_{\min}$ (1a) is applicable only at frequencies much less than $f_{\max}$, as shown in Fig. 2. The linear dependence of $T_{\min}$ on frequency is the same as that of the popular Fukui formulas [5]. The Fukui fitting factor K_f is approximately $\sqrt{[4 \cdot T_d/(T_g \cdot g_m \cdot R_{ds})]}$ [3].

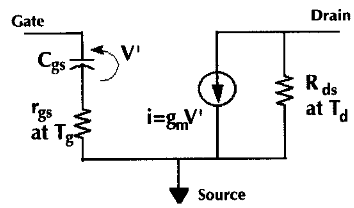

Fig. 1. Equivalent circuit of the intrinsic FET for the noise model.

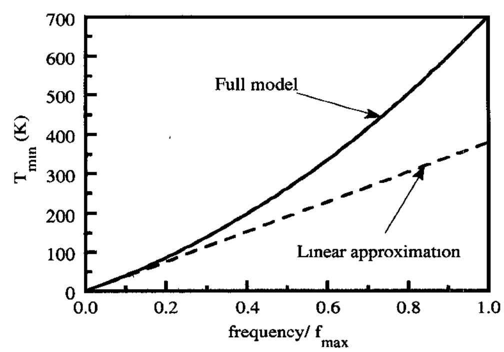

Fig. 2. Plot of $T_{\min}$ and a linear approximation versus frequency normalized to $f_{\max}$ for a FET with a T_g of 298 K and a T_d of 500 K.

The minimum noise figure of a FET, $F_{\min}$, is a simple function of $T_{\min}$:

$$F_{\min} = 10 \log\left(1 + \frac{T_{\min}}{T_0}\right) = \frac{10}{\ln(10)} \ln\left(1 + \frac{T_{\min}}{T_0}\right) \text{dB} \tag{2}$$

$$\approx \frac{10}{\ln(10)} \frac{T_{\min}}{T_0} \text{dB} \quad T_{\min} \ll T_0 \tag{2a}$$

where T_0 is the standard temperature 290 K. The dependence of $F_{\min}$ on $T_{\min}$ is shown in Fig. 3. For $T_{\min}$ much less than

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The author is with the Hewlett-Packard Microwave Technology Division, Santa Rosa, CA 95401.
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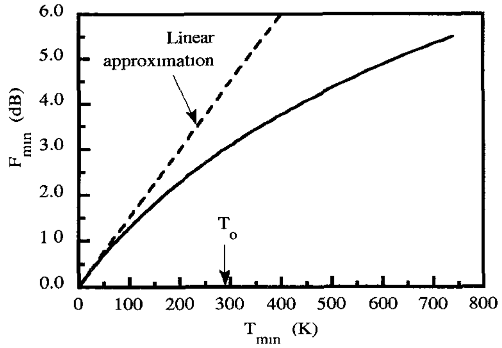

Fig. 3. Plot of F_{min} and a linear approximation versus T_{min}.

T_0, the natural log $(1+X)$ function can be approximated by the first term of a series expansion (2a). F_{min} is proportional to T_{min} only for T_{min} much less than T_0, as shown in Fig. 3.

A simple expression for the frequency dependence of F_{min} is found by substituting the low-frequency approximation for T_{min} (1a) into the approximation for F_{min} (2a). T_g equals αT_0, where α is close to unity.

$$F_{min}(\mathrm{dB}) \approx \frac{10}{\ln(10)}\left(\frac{\alpha T_d}{T_0}\right)^{1/2} \cdot \frac{f}{f_{max}} = \frac{f}{f_n}. \tag{3}$$

The frequency-of-merit f_n is the frequency where F_{min} is 1 dB. A higher f_n is better. f_n is about 20 GHz for a good DBS MODFET. f_n can be used to compare the noise performance of FET's just as f_{max} is used to compare gain performance of FET's.

This simple expression for F_{min} was expected to work only for frequencies much less than f_{max}. However, a comparison of the linear approximation (3) and the full expression for F_{min} in Fig. 4 shows that the linear approximation fits the full expression surprisingly well to frequencies up to f_{max}. The full expression for F_{min} is found by substituting (1) into (2). F_{min} is $5.7(f/f_{max})$ dB for a typical FET with an effective T_d of 500 K. The difference between the linear approximation and the full expression is less than 0.35 dB for frequencies as high as f_{max}, as shown in Fig. 5. For frequencies less than half f_{max}, where the FET is used for an F_{min} less than 3 dB, the difference between the full model and the linear approximations is less than 0.05 dB. The linear dependence of F_{min} on frequency is not unique to Pospieszalski's resistor temperature noise model [2]. This is evident in the F_{min} modeled by Pucel [1].

Why should the simple expression for F_{min} work? T_{min} as a function of frequency increases faster than linear, as shown in Fig. 2. F_{min} as a function of T_{min} increases slower than linear, as shown in Fig. 3. One curve is convex and the other concave. Fortunately, for F_{min} as a function of frequency, the nonlinearities cancel for typical coefficients, and the relationship is almost linear. The linear approximation is better for MODFET's with lower T_d values, such as when the MODFET is biased at lower drain current.

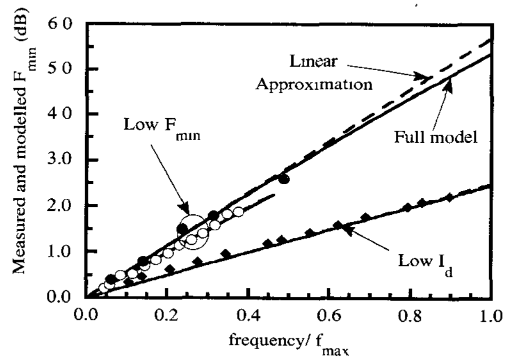

Fig. 4. Plot of F_{min} versus frequency normalized to f_{max} for a FET. The linear approximations (dashed lines) and full models (solid lines) are for MODFET's with a T_g value of 290 K and T_d values of 95 K (low I_{ds}), 372 K, and 500 K (higher I_{ds} for lowest F_{min}). ● are experimental data for 0.25 μm AlGaAs/GaAs MODFET's [6]. ○ and ♦ are measured data for a 0.25 μm pseudomorphic MODFET biased for lowest F_{min} and low I_{ds}, respectively.

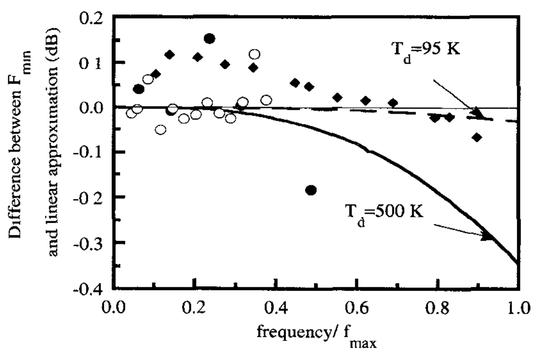

Fig. 5. Plots of the modeled F_{min}'s minus their linear approximations and plots of measured F_{min}'s minus their linear approximations versus frequency normalized to f_{max}. The model and linear approximations are for MODFET's with a T_g value of 290 K and T_d values of 95 K (low I_{ds}) and 500 K (higher I_{ds} for lowest F_{min}). ● are experimental data for 0.25 μm AlGaAs/GaAs MODFET's [6]. ○ and ♦ are measured data for a 0.25 μm pseudomorphic MODFET biased for lowest F_{min} and low I_{ds}, respectively.

III. Experimental Results

Three sets of experimental F_{min} results are shown in Fig. 4. All the sets of F_{min} data are fit well by the linear approximations. The first sets of F_{min} data for 0.25 μm AlGaAs/GaAs MODFET's biased for lowest F_{min} are shown as solid circles in Fig. 4 [6]. These data were chosen for comparison because they spanned a large frequency range (8–62 GHz), and the highest measurement frequency was a higher fraction of f_{max} than other published data. Frequencies were divided by an f_{max} of 127 GHz for the plot of these data. This f_{max} value is extracted from the F_{min} and associated gain data, and agrees well with the f_{max} reported in the paper [3]. Also shown in Fig. 4 are F_{min} data at two biases for a 0.25 μm AlGaAs/InGaAs pseudomorphic MODFET made by Hewlett-Packard. These data were chosen because there was little scatter in the F_{min} data measured at many frequencies, and data were available at two biases. The F_{min} data were measured from 2 to 26.5 GHz with an ATN on-wafer noise measurement system [19]. The second set of F_{min} data for the Hewlett-Packard MODFET biased for lowest F_{min} (I_{ds}/width = 60 mA/mm) is shown by open circles in Fig. 4. Frequencies were divided by an f_{max} of 69 GHz for the plot of these data. The F_{min} data for both MODFET's biased for lowest F_{min} (open and solid circles) show that linear approximation is valid for frequencies to at least half f_{max}.

To demonstrate that the linear approximation worked to frequencies approaching f_{max}, the Hewlett-Packard MODFET was biased to a low drain current (I_{ds}/width = 4 mA/mm). The f_{max} of the 0.25 μm MODFET decreased to 31 GHz at the low drain current bias. Therefore, f_{max} was close to the maximum measurement frequency of 26.5 GHz. F_{min} increased linearly with frequency to frequencies approaching f_{max}, as shown in Fig. 4 as solid diamonds. A least square fit to these data had an excellent fitting coefficient of 0.999. The least squares fit had a nonzero intercept of 0.02 dB that can easily be attributed to measurement error (e.g., noise source ENR uncertainty). The noise data at low drain current were fit with a lower effective T_d of 97 K. T_d is a *fitting factor* for the extrinsic FET noise model, and its value does not have to be physically reasonable. The F_{min} data at low drain current clearly show that linear approximation for F_{min} works well to frequencies approaching f_{max}.

The differences between the measured F_{min} data and the linear approximations are shown in Fig. 5. The points are distributed fairly evenly around zero. The experimental data were fit with the simple linear approximation, $+/-0.2$ dB, to frequencies approaching f_{max}. The measurement uncertainty appeared to be similar to the maximum difference between the linear approximation and the full noise model.

IV. Discussion

The linear expression was compared to published data. Most papers that report F_{min} at millimeter-wave frequencies gave only one or two data points, so they could not be used to demonstrate the frequency dependence of F_{min}. However, there were examples of F_{min} data following the linear approximation [20]. Data sheets for commercial low-noise FET's gave F_{min}'s at many frequencies. In general, F_{min} increased linearly with frequency, as shown in Fig. 6 [7]–[11]. The data demonstrated that the linear approximation worked for low F_{min} values and frequencies much less than f_{max}. Sometimes, deviations from linear frequency dependence were seen at high frequencies. This may be because the measurement uncertainties were worse. For some MODFET's, F_{min} did not decrease with decreasing frequency at low frequencies (e.g., less than 4 GHz), as shown in Fig. 6 [8], [9]. This behavior was not predicted by any of the standard FET noise models. This problem was more evident in some InP-based MODFET's where F_{min} increased with decreasing frequency for some devices [12]. The author is unfamiliar with papers that explain this behavior. This behavior was probably due to noise from gate–drain leakage current or possibly measurement errors because the FET optimum reflection coefficient for minimum noise is large and the FET S_{11} is large [13]. The deviation of F_{min} from linear frequency dependence might be used to identify measurement problems or FET's with nonstandard noise mechanisms.

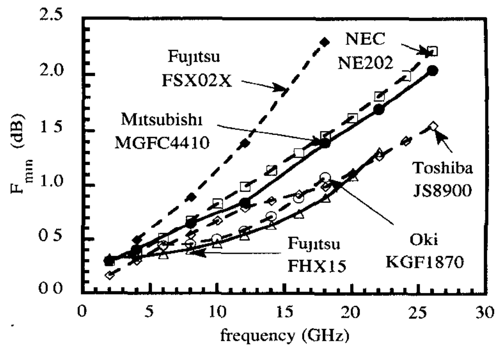

Fig. 6. Plot of F_{min} versus frequency for commercial low-noise MODFET's [7]–[11].

F_{min} is easily extrapolated to higher frequencies with the linear approximation. For example, the Fujitsu SuperHEMT has an F_{min} of 0.55 dB at 12 GHz [8]; the linear approximation predicts an F_{min} of 0.825 dB at 18 GHz. The F_{min} measured at 18 GHz was very close, 0.075 dB higher. For the Fujitsu SuperHEMT example, the frequency-of-merit f_n is 21.8 GHz. F_{min} is simply f/f_n dB.

The approximate expressions for noise parameters assumed by Fukui have been popular with experimentalists because they are simple and they predict the dependence of F_{min} on FET circuit elements and physical parameters [5], [14]. These dependencies were demonstrated with experimental data. Fukui's expression for F_{min} has the form $1 + cf$, where c is a constant. This expression implied that T_{min} was linearly proportional to frequency f. This dependence was not demonstrated. However, experimentalists have successfully fitted F_{min} data versus frequency with the Fukui expression [6], [15]–[18]. Their data are fit equally well with this paper's linear approximation. The maximum error for any measurement was less than 0.16 dB for both expressions. This small error was easily attributed to measurement uncertainties, and illustrated the high quality of these difficult millimeter-wave noise measurements.

V. Conclusions

The simple approximation—minimum noise figure F_{min} increases linearly with frequency—was derived and shown to have an accuracy approaching the measurement uncertainty for many low-noise FET's. Deviations of F_{min} from linear frequency dependence might be used to identify measurement problems or FET's with nonstandard noise mechanisms. F_{min} results for FET's measured at different frequencies are easily compared with the frequency-of-merit f_n, which is f/F_{min}. A higher f_n is better. f_n can be used to compare the noise performance of FET's just as f_T and f_{max} are used to compare the gain performance of FET's.

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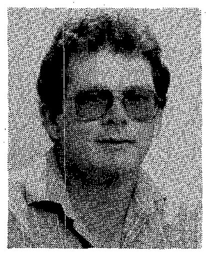

Brian Hughes (M'89) received the B.Sc. (Hons.) degree in material science from Queen Mary College, University of London, and the Ph.D. degree in material science from the University of Southern California, Los Angeles. His dissertation was entitled, "A Transmission Electron Microscopy Study of Ion Implanted GaAs."

He joined the Hewlett-Packard Microwave Technology Division, Santa Rosa, CA, in 1979, to investigate why undoped GaAs was semi-insulating. Next, he gained insights into the origins of flicker noise and reduced the $1/f$ noise corner of GaAs MESFET's to 1 MHz. He helped develop a theory and method for predicting phase noise in microwave oscillators. He was a Visiting Scientist at Cornell University with Prof. Eastman's group in 1988. He was an HP Faculty Loan professor at the University of California, Santa Barbara, in 1990, where he taught a course entitled, "Design and Characterization of High Frequency Devices." He has been involved with the design and characterization of millimeter-wave pseudomorphic, power MODFET's. His current interests are techniques for measuring power and noise figure of FET's and extracting their models. He is currently designing low-noise MODFET MMIC amplifiers.